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**通訊與導航工程系碩士班**  
**數位通訊**  
**Digital Communications**  
**Fall 2004**

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Lecture 12: Trellis Coded Modulation


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## Announcement

- Course webpage:  
<http://dcstl.cge.ntou.edu.tw/DCSTL/Web/dicomm.htm>
- Reading Assignment:
  - ◆ Chapter 8
- No Homework today




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## Turbo Encoder

- Turbo codes, introduced in June 1993, represent the most recent successful attempt in achieving Shannon's theoretical limit.

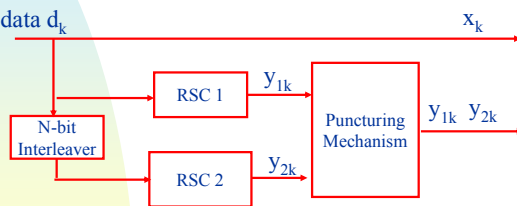




Fig. 2: A turbo encoder




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## Turbo Decoder

- Two component decoders are linked by interleavers in a structure similar to that of the encoder.

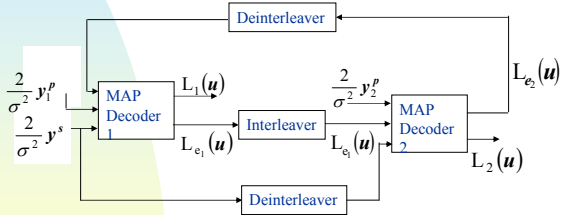



Fig. 3: Turbo Decoder



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## Turbo Decoder

- Each decoder takes three types of soft inputs
  - The received noisy information sequence.
  - The received noisy parity sequence transmitted from the associated component encoder.
  - The a priori information, which is the extrinsic information provided by the other component decoder from the previous step of decoding process.

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## Turbo Decoder

- The soft outputs generated by each constituent decoder also consist of three components:
  - A weighted version of the received information sequence
  - The a priori value, i.e. the previous extrinsic information
  - A newly generated extrinsic information, which is then provided as a priori for the next step of decoding.

$$L_1^{(i)}(\hat{u}_k) = L_{e_2}^{(i-1)}(\hat{u}_k) + \frac{2}{\sigma^2} y_k^s + L_{e_1}^{(i)}(\hat{u}_k) \quad (1)$$

$$L_2^{(i)}(\hat{u}_k) = L_{e_1}^{(i)}(\hat{u}_k) + \frac{2}{\sigma^2} y_k^s + L_{e_2}^{(i)}(\hat{u}_k) \quad (2)$$

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## Turbo Decoder

- The turbo decoder operates iteratively with ever-updating extrinsic information to be exchanged between the two decoder until a reliable hard decision can be made.
- Often, a fixed number, say M, is chosen and each frame is decoded for M iterations.
- Usually, M is set with the worst corrupted frames in mind.
- Most frames, however, need fewer iterations to converge
- It is therefore important to terminate the iterations for each individual frame immediately after the bits are correctly estimated

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## Trellis Coded Modulation

- Conventional error-correcting codes:**
  - Block codes, convolutional codes.
  - Insert extra symbols to add redundancy to signal.
  - Require either expanded bandwidth or lower data rate.
- Trellis Coded Modulation [Ungerboeck, 1982]:**
  - Add redundancy by expanding size of signal set.
  - Combine coding and modulation into a single operation.
  - No loss in rate or bandwidth expansion required.
- Applications:**
  - Telephone modems, microwave, satellite

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### Description of Trellis Codes

Let  $S = \{s_1, \dots, s_O\}$  be some signal constellation. Then the trellis code is specified as follows:

- The transmitted signal is a function of the current state and current input bits
- The next state is a function of the current state and current input bits
- $p_i$  = index of signal point selected by trellis codes
- $p = \{p_i, i = -\infty, \infty\}$  = path followed by encoder
- $C = \{p: p \text{ is a valid path through the trellis}\}$

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### Uncoded 4-PSK and Trellis coded 8-PSK Modulation

(a) One-state trellis

(b) Four-state trellis

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### Four-state TCM with 8-PSK signal Constellation

(a) Encoder

(b) Four-state trellis

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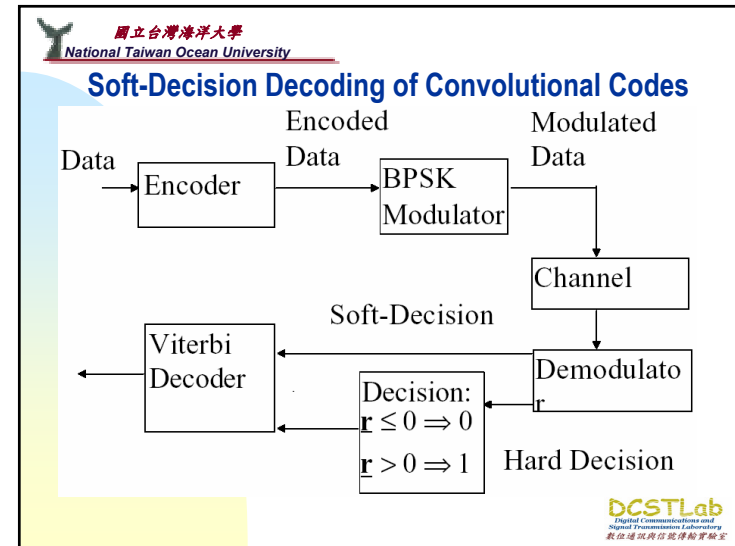
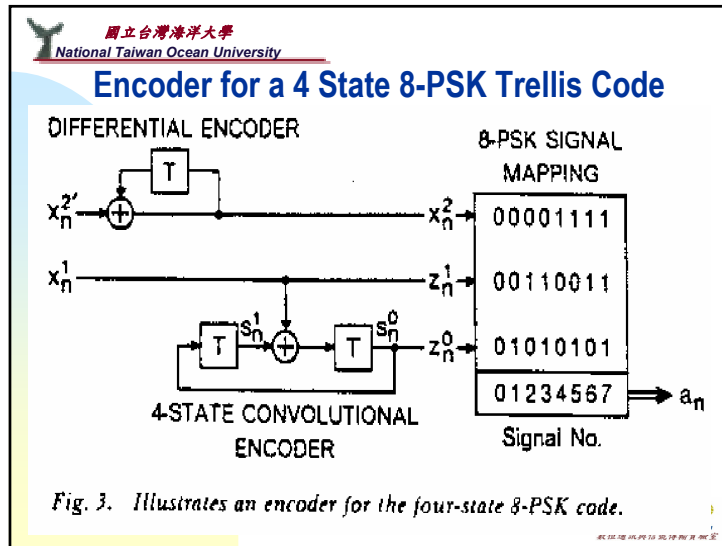
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### Four-state TCM with 8-PSK signal Constellation

(c) Mapping of bits to state transitions

(d) Mapping of bits  $(c_3, c_2, c_1)$  to signal points corresponding to partition in Fig. 8.3-1 (note nonuniqueness of this mapping)

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### Performance of Trellis Codes

- Rate: The rate of the trellis code is the number of bits transmitted per modulation symbol (can be > 1).
- Complexity: proportional to the number of states
- Euclidean Distance Between Signal Points:
 
$$x, y \in S$$

$$d^2(x, y) = \|x - y\|^2$$
- Euclidean Distance Between Paths:
 
$$p, \tilde{p} \in C$$

$$d_{p, \tilde{p}}^2 = \sum_{i=-\infty}^{\infty} d^2(c_{p_i}, c_{\tilde{p}_i})$$

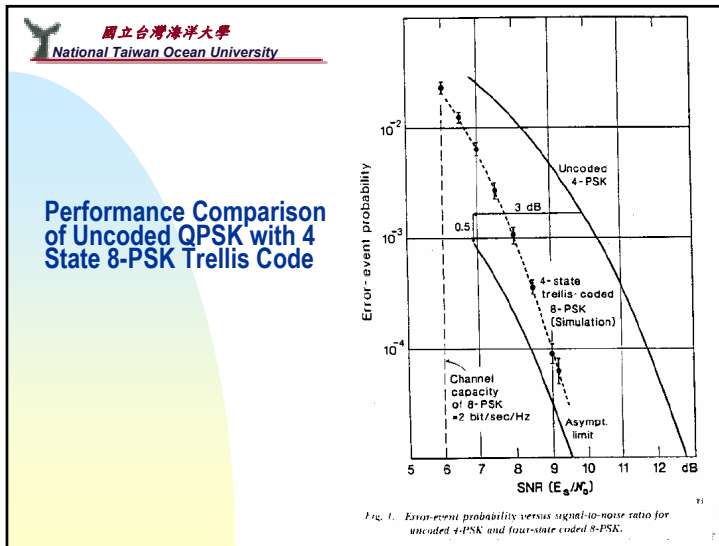
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### Performance of Trellis Codes

- Free Euclidean Distance:
 
$$d_{free}^2 = \min_{\{p, \tilde{p} \in C: p \neq \tilde{p}\}} d_{p, \tilde{p}}^2$$
- Coding Gain:
 
$$G_C = 10 \log_{10} (d_{free}^2 / d_{uncoded}^2)$$
- Error Event Probability:
 
$$P_e \approx a_{d_{free}} Q\left(\frac{d_{free}}{\sqrt{2N_0}}\right)$$

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### References on Trellis Coding

- G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. on Information Theory*, pp. 55-67, Jan. 1982
- G. Ungerboeck, "Trellis coded modulation with redundant signal sets - Parts I&II" *IEEE Communications Magazine*, pp. 5-21, Feb. 1987
- Shuzo Kato, Masahiro Morikura, Shuji Kubota, "Implementation of Coded Modems," *IEEE Comm. Magazine*, pp. 88-97, Dec. 1991.

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- ### Applications of Trellis Coding
- Voice Band Telephone Modems (V.32 standards employ 128 QAM and 32 QAM)
  - Terrestrial Microwave Systems (256 QAM)
  - Mobile Satellite (8 PSK) (Can tolerate slow fading)
  - Few current applications to mobile radio (because of rapid fading)
    - ◆ 'Space-Time Codes' may be thought of as trellis codes, constructed across multiple antenna elements
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